

Derivation of the RQ-equation

Douglas bag gas collection and analysis allow measurement of the respiratory exchange ratio (RER), which in equilibrium should be equal to the respiratory quotient (RQ). The former is related to uptake (V_{O_2}) and elimination (V_{CO_2}) of oxygen and carbon dioxide.

$$\text{Equation 1} \quad RQ = RER = \frac{V_{CO_2}}{V_{O_2}}$$

These in turn are determined by inspired and expired tidal volumes and mixed expired volume fractions of the gases (assuming that inspired gas is free from CO_2):

$$\text{Equation 2} \quad V_{CO_2} = V_e \cdot F_{emixCO_2}$$

$$\text{Equation 3} \quad V_{O_2} = V_i \cdot F_{iO_2} - V_e \cdot F_{emixO_2}$$

The relation between inspired and expired volumes needs to be very accurately determined and to this end nitrogen (N_2 , here also implicitly including Argon) may be used as an inert balance gas with zero net exchange:

$$\text{Equation 4} \quad V_{N_2} = V_i \cdot F_{iN_2} - V_e \cdot F_{emixN_2} = 0$$

Thus, based on this approach (often referred to as the Haldane transformation) inspired volume can be related to expired volume and the balance gas concentrations:

$$\text{Equation 5} \quad V_i = V_e \cdot \frac{F_{emixN_2}}{F_{iN_2}}$$

The nitrogen fractions are given by

$$\text{Equation 6} \quad F_{iN_2} = 1 - F_{iO_2}$$

$$\text{Equation 7} \quad F_{emixN_2} = 1 - F_{emixO_2} - F_{emixCO_2}$$

Substitution of all these relations into the RQ expression (Equation 1) yields after some algebraic simplifications

$$\text{Equation 8} \quad RQ = \frac{(1 - F_{iO_2}) \cdot F_{emixCO_2}}{(1 - F_{emixCO_2}) \cdot F_{iO_2} - F_{emixO_2}}$$

In the experiment, the gas collected in the Douglas bag also contains exhaled water vapour, not included in the equations above, that expands the volume and dilutes the concentration of the other gas substances. If the vapour somehow were to be eliminated, then the equations that are formulated for dry gases would hold. This is in fact what effectively happens when gas is sampled and dried before entering the analysis chamber of the side-stream gas analyser system. Thus, if gas concentrations are reported in a dry state (ATPD, Ambient temperature and pressure, dry) then Equation 8 can be used to calculate RQ.