|   |  | The Sample size ranges from 8   | The Sample size ranges from 16 to   | The Sample size ranges from 36 to  | The Sample size ranges from 73 to  |
|---|--|---|---|--|--|
|   |  | to 15.  | 35 <b>.</b>   | 72.  | 120 and beyond.  |
| Approximation of the mean. Each number in this table represents the average relative error of estimating the sample mean in 200 samples from a Beta distribution.               | LEGEND: The median approximation is represented by black crosses, Formula (4) is represented by blue boxes, and Formula (5) by                                 | 0.035   | 0.034   | 0.044<br>0.042<br>0.044<br>0.036<br>0.036<br>0.034   | 0.045  |
|   | red diamonds.  | 0.015- ° 10 12 14 ×   | 0.024 0 0.024 | 0.028  | 70 80 90 100 110 120   |
|   | Median   | 3.73 %  | 3.21 %  | 2.88 %   | 2.74 %   |
|   | Formula (4)  | 2.46 %  | 3.03 %  | 3.91 %   | 4.70 %   |
|   | Formula (5)  | 2.23 %  | 2.86 %  | 3.80 %   | 4.62 %   |
|   | Conclusion   | Formula (5) is within 2% of the actual sample mean and is performing the best. Formula (4)is almost indistinguishable from Formula (5), and the median is very close behind | All three of these formulas a very close, but the median approximation starts being the best when the sample size reaches about 30.   | The median continues to be the best estimator, separating itself from the other two formulas for the sample sizes in this range.   | The averages stabilize and remain fairly steady as the sample size increases.  |
| Approximation of the standard deviation. Each number in this table represents the average relative error of estimating the sample mean in 200 samples from a Beta distribution. | LEGEND: The Formula (12) is shown using black crosses, Formula (16) is represented by blue boxes; Range/4 by the green circles, and Range/6 by brown diamonds. | 0.4<br>0.1<br>0.1<br>0.1<br>0.1<br>0.1<br>0.1<br>0.1<br>0.1<br>0.1<br>0.1   | 0.35<br>0.25<br>0.15<br>0.1<br>15 20 25 30 35   | 0.35<br>0.35<br>0.25<br>0.15<br>0.16<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10<br>0.10 | 0.45  0.46  0.47  0.47  0.48  0.49 |
|   | Formula (12)   | 10.47 %   | 12.77 %   | 17.44 %  | 23.19 %  |
|   | Formula (16)   | 8.87 %  | 16.32 %   | 29.15 %  | 39.41 %  |
|   | Range/4  | 16.12 %   | 8.91 %  | 12.09 %  | 19.33 %  |
|   | Range/6  | 43.75 %   | 34.21 %   | 26.34 %  | 20.56 %  |
|   | Conclusion   | Formula (16) is the best estimator of the standard deviation in this range of sample sizes.   | The Range/4 formula takes over as the best estimate for the variance  | The Range/4 formula is slowly losing its advantage, and the Range/6 formula is closing in.   | The Range/6 takes over the lead in accuracy when the sample size reaches 100, and keeps it as the sample sizes increase.   |

TABLE 3: Beta Distribution with parameters a = 9 and b = 4.