

Skew Normal Distribution

A random variable Z follows a Skew-Normal distribution if its probability density function (pdf) is:

$$f(z) = \frac{2}{\sqrt{\sigma^2 + \delta^2}} \phi\left(\frac{z - \mu}{\sigma^2 + \delta^2}\right) \Phi\left(\frac{\delta}{\sigma} \frac{z - \mu}{\sigma^2 + \delta^2}\right), \quad z \in R$$

where ϕ and Φ are the standard normal pdf and cdf(cumulative distribution function) respectively. δ is referred to as the skewness parameter, when δ is zero $Z \sim N(\mu, \sigma^2)$.

Expected value of Z and its variance are as follows:

$$E(Z) = \mu + \delta \sqrt{2/\pi} \quad \text{and} \quad \text{Var}(Z) = \sigma^2 + \delta^2 \left(1 - \frac{2}{\pi}\right)$$