

Table Additional file 2: Table S1. Simulation study results based on 1000 data sets ($n = 250$ and 1000). Means of all estimates along with their corresponding 2.5th and 97.5th percentile values within parentheses. The theoretical MSE is $1.74^2 = 3.028$.

Estimates	Without missing data						With missing data					
	$n = 250$						$n = 1000$					
	best	tol		best	tol		best	tol		best	tol	
$MSE_{apparent}$	2.862(2.347;3.427)	3.152(2.588;3.799)					2.782(2.121;3.411)	3.041(2.289;3.770)				
$MSE_{external}$	3.272(2.687;3.929)	3.476(2.858;4.234)					3.450(2.787;4.210)	3.508(2.866;4.315)				
$Optimism_{external}$	-0.411(-1.199;0.357)	-0.324(-1.205;0.538)					-0.668(-1.699;0.182)	-0.467(-1.537;0.427)				
β_{LP}	1.063(1.042;1.088)	1.162(1.123;1.206)					1.042(1.018;1.067)	1.138(1.092;1.186)				
	best	tol		Appr 1 best	Appr 1 tol		Appr 2 best	Appr 2 tol		Appr 3 best	Appr 3 tol	
$Optimism_{internal}$	-0.421(-0.521;-0.334)	-0.344(-0.442;-0.264)		-0.343(-0.444 -0.243)	-0.301(-0.393;-0.220)		-0.041(-0.060;-0.024)	-0.034(-0.054;-0.017)		-0.421(-0.540;-0.307)	-0.354(-0.461;-0.258)	
$MSE_{corrected}$	3.283(2.746;3.868)	3.496(2.927;4.143)		3.124(2.422;3.795)	3.342(2.551;4.084)		2.822(2.159;3.472)	3.075(2.309;3.813)		3.203(2.486;3.864)	3.395(2.610;4.157)	
$\bar{\beta}_{LP^*}$	1.028(1.013;1.041)	1.124(1.097;1.155)		1.017(1.001;1.033)	1.109(1.072;1.145)		1.034(1.016;1.052)	1.127(1.087;1.170)		1.018(1.003;1.038)	1.107(1.067;1.151)	
	best	tol		Appr 1 best	Appr 1 tol		Appr 2 best	Appr 2 tol		Appr 3 best	Appr 3 tol	
$Optimism_{internal}$	-0.109(-0.138;-0.082)	-0.076(-0.105;-0.048)		-0.091(-0.119;-0.065)	-0.067(-0.093;-0.039)		-0.011(-0.019;-0.003)	-0.008(-0.017;0.001)		-0.111(-0.141;-0.081)	-0.078(-0.107;-0.048)	
$MSE_{corrected}$	3.096(2.828;3.381)	3.268(2.985;3.564)		3.063(2.748;3.395)	3.241(2.903;3.597)		2.983(2.676;3.297)	3.182(2.846;3.523)		3.083(2.761;3.422)	3.252(2.910;3.603)	
$\bar{\beta}_{LP^*}$	1.020(1.014;1.026)	1.125(1.111;1.140)		1.018(1.013;1.022)	1.119(1.103;1.135)		1.022(1.019;1.025)	1.123(1.106;1.140)		1.018(1.013;1.024)	1.119(1.100;1.137)	
	best	tol		Appr 1 best	Appr 1 tol		Appr 2 best	Appr 2 tol		Appr 3 best	Appr 3 tol	

"Appr 1", "Appr 2", "Appr 3" and "Appr 4" are respectively the four approaches to handle missing data.

"best" and "tol" stand for the model with optimal penalty value and that within 3% of the optimal model respectively

$MSE_{apparent}$ was the performance on the original data

$MSE_{external}$ was the performance on an independent new data with no missing values

$Optimism_{internal}$ was the difference between the bootstrap performance (on bootstrap data) and the test performance (on original data)

$Optimism_{external} = MSE_{apparent} - MSE_{external}$

$MSE_{corrected} = MSE_{apparent} - Optimism_{internal}$

β_{LP} was the slope of the linear predictor (LP) estimated from regressing the observed outcome on LP from the original data

$\bar{\beta}_{LP^*}$ was the slope of LP^* estimated by regressing the outcome in the original sample on the LP^* from the bootstrap sample.