Additional file 3. A proof that the bounds are sharp.

To prove that the bounds on sufficient-cause interactions as presented in text are indeed sharp bounds, it suffices to show that all the inequalities in Web Appendix 1 are attainable:

A1.8 can be attained when

$$\text{Rate}_{\text{class}=i',j}(t) = \text{Rate}_{\text{class}=i,j'}(t) = \text{Rate}_{\text{class}=i',*}(t) = \text{Rate}_{\text{class}=*,j'}(t) = \text{Rate}_{\text{class}=*,*}(t) = 0$$

for $(i' \neq i) \in \{1,...,L_1\}$ and $(j' \neq j) \in \{1,...,L_2\}$;

A1.10 and A1.14 can be attained when

$$\text{Rate}_{\text{class}=i,*}(t) = \text{Rate}_{\text{class}=*,j}(t) = \text{Rate}_{\text{class}=*,*}(t) = 0 \quad \text{for} \quad i \in \{1,...,L_1\} \quad \text{and} \quad j \in \{1,...,L_2\};$$

A1.12 can be attained when

$$\text{Rate}_{\text{class}=i,j}(t) = 0 \quad \text{if} \quad u_i \times v_j \neq 1.$$