

Appendix

Calculation of the sample size formula

When combining (6) and (7) and replacing $\Delta_j^2|r_j$ by it's expectation, the following calculations lead to the overall sample size formula (9).

$$\begin{aligned} (q_{1-\alpha/2} + q_{1-\beta})^2 &= \frac{\mu^2}{\text{Var}(\mu)} \\ \Leftrightarrow N^2 - N \frac{\sigma^2(k+1)^2}{k} \left(\frac{q_{1-\alpha/2} + q_{1-\beta}}{\mu} \right)^2 \\ &= \tau^2(k+1)^2 \sum_{j=1}^c \text{E}(\Delta_j^2|r_j) \left(\frac{q_{1-\alpha/2} + q_{1-\beta}}{\mu} \right)^2 \end{aligned}$$

This quadratic equation can be solved with standard algebra techniques and leads to the following result

$$\begin{aligned} N_{1,2} &= \frac{\sigma^2(k+1)^2}{2k} \left(\frac{q_{1-\alpha/2} + q_{1-\beta}}{\mu} \right)^2 \\ &\pm \sqrt{\frac{\sigma^4(k+1)^4}{4k^2} + \frac{\tau^2(k+1)^2\mu^2 \sum_{j=1}^c \text{E}(\Delta_j^2|r_j)}{(q_{1-\alpha/2} + q_{1-\beta})^2}} \\ &\cdot \left(\frac{q_{1-\alpha/2} + q_{1-\beta}}{\mu} \right)^2. \end{aligned}$$

Since we are interested in positive integers, we take the positive solution from above. \square