

Additional File 1: An equivalence between pointwise confidence bands for $\theta(\cdot)$ and $\theta^{-1}(\cdot)$

Let $\text{CI}(x)$ denote a $1 - \gamma$ pointwise confidence band for $\theta(x)$. For any η of the target space of $\theta(\cdot)$ then $\varphi_{\eta,x} := \mathbb{I}(\eta \notin \text{CI}(x))$ defines a level γ test for $H_0 : \theta(x) = \eta$ and we have $\text{CI}(x) = \{\eta \mid \varphi_{\eta,x} = 0\}$. Now $\theta(x) = \eta$ is equivalent to $x \in \theta^{-1}(\eta)$. Consequently, $\varphi_{\eta,x}$ is a level γ test for $H_0 : x \in \theta^{-1}(\eta)$. Hence $\tilde{\text{CI}}(\eta) := \{x \mid \varphi_{\eta,x} = 0\}$ is a confidence band for $\theta^{-1}(\eta)$ in the sense that for any $x \in \theta^{-1}(\eta)$ it holds that $P(x \in \tilde{\text{CI}}(\eta)) \geq 1 - \gamma$.