Additional File 3: Calculation of quality measures

Let f_{X^*} be the density function of X^*. Let $C_T = \{x \mid \theta(x) \geq 0\}$. Then we can express sensitivity, specificity and overall gain as

$$\begin{aligned}
\text{Sensitivity} =& P\Big(X^* \in C \mid \theta(X^*) \geq 0\Big) = P\Big(X^* \in C \cap \theta(X^* \geq 0)\Big) \Big/ P\Big(\theta(X^* \geq 0)\Big) \\
=& \int\limits_{C \cap C_T} f_{X^*}(x)\,dx \Big/ \int\limits_{C_T} f_{X^*}(x)\,dx \\
\text{Specificity} =& P\Big(X^* \notin C \mid \theta(X^*) < 0\Big) = P\Big(X^* \notin C \cap \theta(X^*) < 0\Big) \Big/ P\Big(\theta(X^*) < 0\Big) \\
=& \int\limits_{\overline{C} \cap \overline{C_T}} f_{X^*}(x)\,dx \Big/ \int\limits_{\overline{C_T}} f_{X^*}(x)\,dx \\
\text{Overall gain} =& E(\theta(X^*)\mathbb{1}_{X^* \in C}) = \int\limits_{C} \theta(x) f_{X^*}(x)\,dx
\end{aligned}$$

Since for all construction methods and all choices of $\theta(x)$ the sets C, C_T and their complements can be written as the union of a finite number of disjunct intervals, we need only to consider how to compute $\int\limits_{(a,b)} f_{X^*}(x)\,dx$ and $\int\limits_{(a,b)} \theta(x) f_{X^*}(x)\,dx$. For the special cases of $\theta(x)$ to be linear or quadratic, we can derive explicit formulas for these integrals; the corresponding formulas are shown in the following table:

	$\int\limits_{(a,b)} f_{X^*}(x)\,dx$	$\int\limits_{(a,b)} \theta(x) f_{X^*}(x)\,dx$
$X \sim U(0,1)$	$b-a$	
$\theta(x) = \beta(x-0.5)$		$\beta/2((b^2-a^2)-(b-a))$
$\theta(x) = \beta(l+m(x+n)^2)$		$\beta((l+mn^2)(b-a)$ $+mn(b^2-a^2)+m/3(b^3-a^3))$
$X \sim T(0,1,1/3)$		
$\theta(x) = \beta(x-0.5)$		
case 1: $1/3 < a < b$	$3(b-a)-3/2(b^2-a^2)$	$\beta(-3/2(b-a)+9/4(b^2-a^2)-(b^3-a^3))$
case 2: $a < b \leq 1/3$	$3(b^2-a^2)$	$\beta(2(b^3-a^3)-3/2(b^2-a^2))$
case 3: $a \leq 1/3 < b$	$3((1/9-a^2)+(b-1/3)$ $-1/2(b^2-1/9))$	$\beta(7/36-2a^3+3/2a^2-3/2b+9/4b^2-b^3)$