Additional file 1

Calculation of the basic reproduction number:
System (2.2) can be rewritten as follows,

$$\begin{cases} \dot{s}_k^{h1} = -\beta_h k s_k^{h1}\Theta_{11} - \beta_{ah1}\frac{s_k^{h1}N^m}{N^h} i^a \\ \dot{e}_k^{h1} = \beta_h k s_k^{h1}\Theta_{11} + \beta_h k s_k^{h2}\Theta_{22} + \beta_{ah1}\frac{s_k^{h1}N^m}{N^h} i^a - \lambda_h e_k^{h1}, \\ \dot{p}_k^{h1} = \lambda_h e_k^{h1} - \eta_h p_k^{h1}, \\ \dot{i}_k^{h1} = \eta_h p_k^{h1} - (\gamma_h + d_h)\, i_k^{h1}, \\ \dot{r}_k^{h1} = \gamma_h i_k^{h1}, \\ \dot{s}_k^{h2} = -\beta_h k s_k^{h2}\Theta_{22} - \beta_{ah2}\frac{s_k^{h2}N^m}{N^h} i^a, \\ \dot{e}_k^{h2} = \beta_{ah2}\frac{s_k^{h2}N^m}{N^h} i^a - \lambda_h e_k^{h2}, \\ \dot{p}_k^{h2} = \lambda_h e_k^{h2} - \eta_h p_k^{h2}, \\ \dot{i}_k^{h2} = \eta_h p_k^{h2} - (\gamma_h + d_h)\, i_k^{h2}, \\ \dot{r}_k^{h2} = \gamma_h i_k^{h2}, \\ \dot{s}^a = -\beta_a s^a i^a \\ \dot{e}^a = \beta_a s^a i^a - \lambda_a e^a, \\ \dot{i}^a = \lambda_a e^a - (\gamma_a + d_a)\, i^a, \\ \dot{r}^a = \gamma_a i^a. \end{cases}$$

Here,

$$\Theta_{11} = \frac{c_{hh}\sum\limits_{k=1}^{n} kp(k)\left(\xi_1 p_k^{h1} + i_k^{h1}\right)}{\langle k\rangle}, \Theta_{22} = \frac{c_{lh}\sum\limits_{k=1}^{n} kp(k)\left(\xi_1 p_k^{h1} + i_k^{h1}\right)}{\langle k\rangle}.$$

The above system is an $10n + 4$ dimension system, where $s_k^{h1}(t) = S_k^{h1}(t)/N_k^{h1}$ means the relative density of susceptible individuals with degree k at time t in high-risk groups, $e_k^{h1}(t), p_k^{h1}(t), i_k^{h1}(t), r_k^{h1}(t), s_k^{h2}(t), e_k^{h2}(t)$, $p_k^{h2}(t), i_k^{h2}(t), r_k^{h2}(t), s^a(t), e^a(t), i^a(t), r^a(t)$ are similar to $s_k^{h1}(t)$. We quantify the total number of infected individuals by calculating epidemic thresholds to determine whether an epidemic will spread or die out. To estimate the transmission potential of the epidemic, we derive R_0 of the model, important epidemic threshold, defined as the average secondary cases produced by an infected individual in a completely susceptible population [1]. There exists a unique disease-free equilibrium,

$$
\begin{aligned}
&(s_1^{h1},\dots,s_n^{h1},E_1^{h1},\dots,e_n^{h1},p_1^{h1},\dots,p_n^{h1},i_1^{h1},\dots,i_n^{h1},r_1^{h1},\dots,r_n^{h1},s_1^{h2},\dots,\\
&s_n^{h2},e_1^{h2},\dots,e_n^{h2},p_1^{h2},\dots,p_n^{h2},i_1^{h2},\dots,i_n^{h2},r_1^{h2},\dots,r_n^{h2},s^a,e^a,i^a,r^a)\\
&=(1,\dots,1,0,\dots,0,0,\dots,0,0,\dots,0,0,\dots,0,1,\dots,1,\\
&0,\dots,0,0,\dots,0,0,\dots,0,0,\dots,0,1,0,0,0)=:P_0.
\end{aligned}
$$

We compute R_0 using the next generation matrix approach [2, 3]. The Jacobian matrix F is

$$
F=\begin{bmatrix}
0_{n\times n} & \frac{\beta_h\xi_1(c_{hh}+c_{lh})}{\langle k\rangle}f_1 & \frac{\beta_h(c_{hh}+c_{lh})}{\langle k\rangle}f_1 & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & \beta_{ah1}\frac{N^m}{N^h}I_1\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & \beta_{ah2}\frac{N^m}{N^h}I_1\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0 & \beta_a\\
0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0 & 0
\end{bmatrix},
$$

where I_1 is a unit vector of dimension $n*1$, and

$$
f_1=\begin{pmatrix}
p(1) & 2p(2) & \cdots & np(n)\\
2p(1) & 4p(2) & \cdots & 2np(n)\\
\vdots & \vdots & \ddots & \vdots\\
np(1) & 2np(2) & \cdots & n^2p(n)
\end{pmatrix}.
$$

The matrices V is

$$
V=\begin{bmatrix}
\lambda_hI_n & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
-\lambda_hI_n & \eta_hI_n & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & -\eta_hI_n & (\gamma_h+d_h)I_n & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & \lambda_hI_n & 0_{n\times n} & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & -\lambda_hI_n & \eta_hI_n & 0_{n\times n} & 0_{n\times 1} & 0_{n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & -\eta_hI_n & (\gamma_h+d_h)I_n & 0_{n\times 1} & 0_{n\times 1}\\
0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & \lambda_a & 0\\
0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & -\lambda_a & (\gamma_a+d_a)
\end{bmatrix}.
$$

Therefore,

$$
FV^{-1}=\begin{bmatrix}
A & A & B & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & C & C\\
0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times 1} & 0_{2n\times 1}\\
0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & D & D\\
0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times n} & 0_{2n\times 1} & 0_{2n\times 1}\\
0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & E & E\\
0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0_{1\times n} & 0 & 0
\end{bmatrix},
$$

where

$$A = \frac{\beta_h(c_{hh}+c_{lh})}{\langle k\rangle}\left(\frac{\xi_1}{\eta_h}+\frac{1}{\gamma_h+d_h}\right)f_1, B = \frac{\beta_h(c_{hh}+c_{lh})}{(\gamma_h+d_h)\langle k\rangle}f_1, C = \beta_{ah1}\frac{N^m}{N^h(\gamma_a+d_a)}I_1,$$

$$D = \beta_{ah2}\frac{N^m}{N^h(\gamma_a+d_a)}I_1, E = \frac{\beta_a}{\gamma_a+d_a}.$$

The characteristic equation of FV^{-1} is

$$\begin{aligned}&\left|\lambda I_{(8n+10)\times(8n+10)} - FV^{-1}\right| \\ &= \begin{vmatrix} \lambda I_{n\times n} - A & -A & -B & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & -C & -C \\ 0_{n\times n} & M & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & M & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & M & 0_{n\times n} & 0_{n\times n} & -D & -D \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & M & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & M & 0_{n\times n} & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & \lambda - E & -E \\ 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} & 0 & \lambda \end{vmatrix} = 0,\end{aligned}$$

where

$$M = \begin{pmatrix} \lambda & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda \end{pmatrix}_{(n+1)\times(n+1)}.$$

The characteristic equation of FV^{-1} is equal to

$$\lambda^{5n+1}(\lambda - E)\left|\lambda I_{n\times n} - A\right| = 0,$$

Therefore, the similarity matrix has the same eigenvalue, we have

$$f_1 = \begin{pmatrix} p(1) & 2p(2) & \cdots & np(n) \\ 2p(1) & 4p(2) & \cdots & 2np(n) \\ \vdots & \vdots & \ddots & \vdots \\ np(1) & 2np(2) & \cdots & n^2p(n) \end{pmatrix}_{n\times n} \sim \begin{pmatrix} f_a & 2p(2) & \cdots & np(n) \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{pmatrix}_{n\times n},$$

where

$$f_a = p(1) + 2^2p(2) + \cdots + n^2p(n) = \left\langle k^2\right\rangle.$$

The basic reproduction number R_0 becomes

$$R_0 = \max\left\{\beta_h(c_{hh}+c_{lh})\left(\frac{\xi_1}{\eta_h}+\frac{1}{\gamma_h+d_h}\right)\frac{\left\langle k^2\right\rangle}{\langle k\rangle}, \frac{\beta_a}{\gamma_a+d_a}\right\}.$$

References

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[2] Van den Driessche P, Watmough J. Further notes on the basic reproduction number. Math Epidemiology. 2008;1945:159–178.

[3] Diekmann O, Heesterbeek JAP, Metz JA. On the definition and the computation of the basic reproduction ratio R_0 in models for infectious diseases in heterogeneous populations. J Math Biol. 1990;28:365–382.