The SRMF algorithm

Here let u_i and v_j be the *i*-th and *j*-th row vectors of *U* and *V*, respectively. We take the partial derivative of objective function L, i.e. $\frac{\partial L}{\partial u_i}$ and obtain the updating rule of *U* by setting $\frac{\partial L}{\partial u_i}$ =0, as follows:

$$\frac{\partial L}{\partial u_i}\{\sum_{j=1}^{n} W_{ij} \times (Y_{ij} - u_i v_j^T)]^2 + \lambda_l \sum_{j=1}^{n} u_{ij}^2 + \lambda_d \sum_{p=1}^{m}\sum_{q=1}^{m}[S_d(p,q) - u_p u_q^T]^2\}$$

$$= 2\sum_{j=1}^{n} W_{ij}^2 \times (Y_{ij} - u_i v_j^T) \times (-v_j) + 2\lambda_l u_i + 4\lambda_d \sum_{p=1}^{m}[S_d(p,i) - u_p u_i^T] \times (-u_p)$$

$$= 2\sum_{j=1}^{n} W_{ij}^2 \times (Y_{ij} - u_i v_j^T) \times (-v_j) + 2\lambda_l u_i + 4\lambda_d \sum_{p=1}^{m}[S_d(p,i) - u_i u_p^T] \times (-u_p)$$

$$= 0$$

As W_{ij} = 0 or 1, so

$$u_i = [\sum_{j=1}^{n} W_{ij} Y_{ij} v_j + 2\lambda_d \sum_{p=1}^{m} S_d(p,i) u_p](\sum_{j=1}^{n} W_{ij} v_j^T v_j + \lambda_l I_K + 2\lambda_d \sum_{p=1}^{m} u_p^T u_p)^{-1} \quad (1)$$

Similarly, according to $\frac{\partial L}{\partial b_j}$ =0, we obtain the updating rule of *V* as follows:

$$v_j = [\sum_{i=1}^{m} W_{ij} Y_{ij} u_i + 2\lambda_c \sum_{h=1}^{n} S_c(h,j) v_h](\sum_{i=1}^{m} W_{ij} u_i^T u_i + \lambda_l I_K + 2\lambda_c \sum_{h=1}^{n} v_h^T v_h)^{-1} \quad (2)$$